

Terms and Equations of Reverse Osmosis

There is a set of terms and equations used to define the parameters governing transport across a membrane. This paper presents the concepts of Reverse Osmosis and the equations used to describe them.

OSMOTIC PRESSURE

The osmotic pressure, P_{osm} , of a solution can be determined experimentally by measuring the concentration of dissolved salts in solution :

$$P_{osm} = 1.19 (T + 273) * \Sigma(m_i) \quad (1)$$

where P_{osm} = osmotic pressure (in psi), T is the temperature (in °C), and $\Sigma(m_i)$ is the sum of molal concentration of all constituents in a solution. An approximation for P_{osm} may be made by assuming that 1000 ppm of Total Dissolved Solids (TDS) equals about 11 psi (0.76 bar) of osmotic pressure.

The mechanism of water and salt separation by reverse osmosis is not fully understood. Current scientific thinking suggests two transport models: porosity and diffusion. That is, transport of water through the membrane may be through physical pores present in the membrane (porosity), or by diffusion from one bonding site to another within the membrane. The theory suggests that the chemical nature of the membrane is such that it will absorb and pass water preferentially to dissolved salts at the solid/liquid interface. This may occur by weak chemical bonding of the water to the membrane surface or by dissolution of the water within the membrane structure. Either way, a salt concentration gradient is formed across the solid/liquid interface. The chemical and physical nature of the membrane determines its ability to allow for preferential transport of solvent (water) over solute (salt ions).

WATER TRANSPORT

The rate of water passage through a semipermeable membrane is defined in Equation 2.

$$Q_w = (\Delta P - \Delta P_{osm}) * K_w * S/d \quad (2)$$

where Q_w is the rate of water flow through the membrane, ΔP is the hydraulic pressure differential across the membrane, ΔP_{osm} is the osmotic pressure differential across the membrane, K_w is the membrane permeability coefficient for water, S is the membrane area, and d is the membrane thickness. This equation is often simplified to:

$$Q_w = A * (NDP) \quad (3)$$

Where A represents a unique constant for each membrane material type, and NDP is the net driving pressure or net driving force for the mass transfer of water across the membrane.

SALT TRANSPORT

The rate of salt flow through the membrane is defined by Equation 4:

$$Q_s = \Delta C * K_s * S/d \quad (4)$$

where Q_s is the flow rate of salt through the membrane, K_s is the membrane permeability coefficient for salt, ΔC is the salt concentration differential across the membrane, S is the membrane area, and d is the membrane thickness. This equation is often simplified to:

$$Q_s = B * (\Delta C) \quad (5)$$

Where B represents a unique constant for each membrane type, and ΔC is the driving force for the mass transfer of salts.

Equations 4 and 5 show that for a given membrane:

- a) Rate of water flow through a membrane is proportional to net driving pressure differential (NDP) across the membrane.
- b) Rate of salt flow is proportional to the concentration differential across the membrane and is independent of applied pressure.

Salinity of the permeate, C_p , depends on the relative rates of water and salt transport through reverse osmosis membrane:

$$C_p = Q_s/Q_w \quad (6)$$

The fact that water and salt have different mass transfer rates through a given membrane creates the phenomena of salt rejection. No membrane is ideal in the sense that it absolutely rejects salts; rather the different transport rates create an apparent rejection. The equations 2, 4 and 5 explain important design considerations in RO systems. For example, an increase in operating pressure will increase water flow without changing salt flow, thus resulting in lower permeate salinity.

SALT PASSAGE

Salt passage is defined as the ratio of concentration of salt on the permeate side of the membrane relative to the average feed concentration. Mathematically, it is expressed in Eq. 7:

$$SP = 100\% * (C_p/C_{fm}) \quad (7)$$

where SP is the salt passage (in %), C_p is the salt concentration in the permeate, and C_{fm} is the mean salt concentration in feed stream.

Applying the fundamental equations of water flow and salt flow illustrates some of the basic principles of RO membranes. For example, salt passage is an inverse function of pressure; that is, the salt passage increases as applied pressure decreases. This is because reduced pressure decreases permeate flow rate, and hence, dilution of salt (the salt flows at a constant rate through the membrane as its rate of flow is independent of pressure).

SALT REJECTION

Salt rejection is the opposite of salt passage, and is defined by Equation 8.

$$SR = 100\% - SP \quad (8)$$

where SR is the salt rejection (in %), and SP is the salt passage as defined in Equation 7.

PERMEATE RECOVERY RATE (CONVERSION)

Permeate recovery is another important parameter in the design and operation of RO systems. Recovery or conversion rate of feed water to product (permeate) is defined by Equation 9.

$$R = 100\% * (Q_p/Q_f) \quad (9)$$

where R is recovery rate (in %), Q_p is the product water flow rate, and Q_f is the feed water flow rate. The recovery rate affects salt passage and product flow. As the recovery rate increases, the salt concentration on the feed-brine side of the membrane increases, which causes an increase in salt flow rate across the membrane as indicated by Equation 5. Also, a higher salt concentration in the feed-brine solution increases the osmotic pressure, reducing the NDP and consequently reducing the product water flow rate according to Equation 2.

CONCENTRATION POLARIZATION

As water flows through the membrane and salts are rejected by the membrane, a boundary layer is formed near the membrane surface in which the salt concentration exceeds the salt concentration in the bulk solution. This increase of salt concentration is called concentration polarization. The effect of concentration polarization is to reduce actual product water flow rate and salt rejection versus theoretical estimates. The effects of concentration polarization are as follows:

1. Greater osmotic pressure at the membrane surface than in the bulk feed solution, ΔP_{osm} , and reduced Net Driving Pressure differential across the membrane ($\Delta P - \Delta P_{osm}$).
2. Reduced water flow across membrane (Q_w).
3. Increased salt flow across membrane (Q_s).
4. Increased probability of exceeding solubility of sparingly soluble salts at the membrane surface, and the distinct possibility of precipitation causing membrane scaling.

The Concentration Polarization Factor (CPF) can be defined as a ratio of salt concentration at the membrane surface (C_s) to bulk concentration (C_b).

$$CPF = C_s/C_b \quad (10)$$

An increase in permeate flux will increase the delivery rate of ions to the membrane surface and increase C_s . An increase of feed flow increases turbulence and reduces the thickness of the high concentration layer near the membrane surface. Therefore, the CPF is directly proportional to permeate flow (Q_p), and inversely proportional to average feed flow (Q_{favg}).

$$CPF = K_p * \exp(Q_p / Q_{favg}) \quad (11)$$

Where K_p is a proportionality constant depending on system geometry.

Using the arithmetic average of feed and concentrate flow as average feed flow, the CPF can be expressed as a function of the permeate recovery rate a of membrane element (R_i).

$$CPF = K_p * \exp(2R_i / (2 - R_i)) \quad (12)$$

The value of the Concentration Polarization Factor of 1.20, which is the recommended Hydranautics limit, corresponds to 18% permeate recovery for a 40" long membrane element.